Week 6. Neuron & Hodgkin-Huxley model

· Overview
- Biological background
- modeling (Hodgkin-Huxley model) → circuit theory & electrochemistry
- Coding ← 다음주...

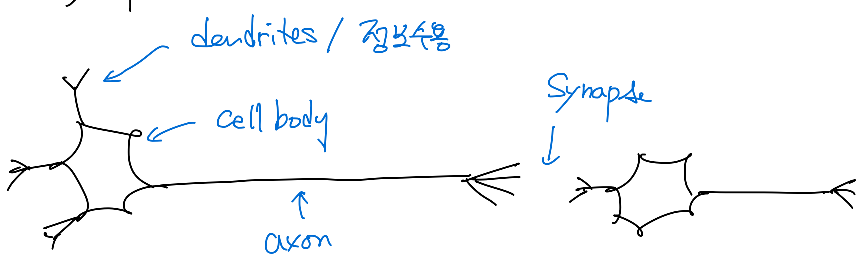
· motivation : Brain is complex system that consists of a lots of "Neurons"

$\boxed{\text{Single Neuron} \rightarrow \text{Neural Network} \rightarrow \text{Brain (?)}}$

· Biological background

Neuron
1. Structure
2. membrane potential (휴지막 전위)
3. Action Potential (활동전위)
4. Synapse

· Structure



input → neuron → output

· membrane potential / 평형전위 = − 70 mV

$\Delta V = V_{내부} - V_{외부}$

0mV

− 70mV

※ $Na^+ - K^+$ → 세포 외부 Na^+ ↑

내부 K^+ ↑

요인
- $Na^+ - K^+$ pump
  - $3Na^+$
  - 외
  - 내
  - $2K^+$
- diffusion / 투과도 leak (∝ 채널 수)
  - Na^+ : K^+
  - 1 : 50
  - 못들어옴 잘나감.
- 세포내부 음전하 단백질 ↑

막

내	외

K^+ ↑ K^+ ↓ : 화학적 기울기

−70mV 0mV : 전기적 기울기

확

확산 (투과 ∝ 채널)

$\boxed{-90mV}$

−90mV $\overset{K^+}{\Longrightarrow}$ 0mV

K^+ 평형전위

내 외

Na^+ ↓ Na^+ ↑

−70mV 0mV

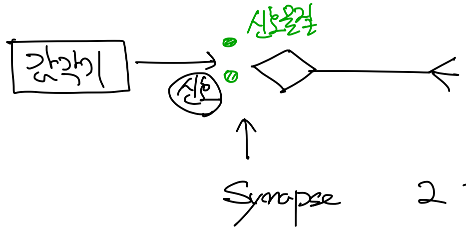
확

전기 + 화학적 기울기

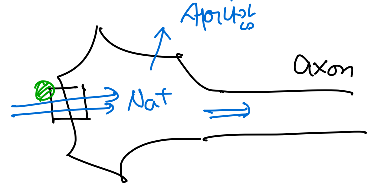
+60mV 0mV

$\boxed{-70mV}$

Action potential



2 →



3 ↑ 시·공간 合

↙ 계속 ↘ 여러곳에서

시간적변화

전

\+

− (분극)

Resting potential

(−70mV)

→

직후

− (탈분극) | +

\+ Depolarized $\boxed{+30mV}$ | 분극 −

Na^+ ↑

Voltage gated Na^+ channel

(−55mV 열림)

→

후

재분극	탈분극	분극
		Na^+

K^+ Na^+

Voltage gated K^+ channel

(+30mV open)

더 후

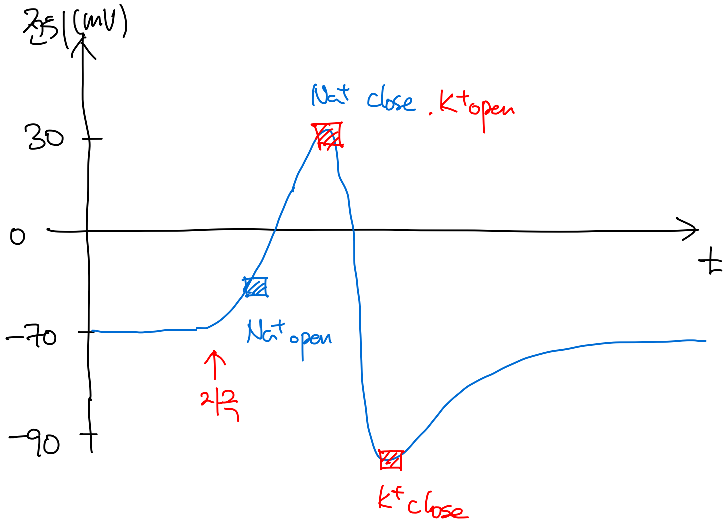
→

과분극 (hyperpol.)	재	탈	분
−90mV		+30mV	−70mV

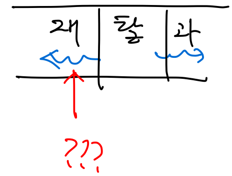
→

더더더후

분	과	재	탈	분
−70mV	−90mV		+30mV	−70mV

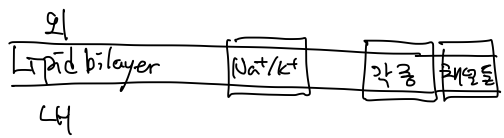


Question

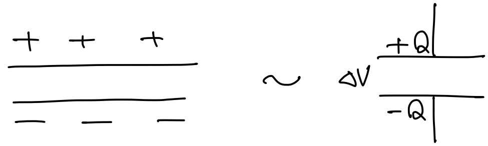


· modeling (H-H model) ← How can we model actionpotential?

- membrane current
- Nernst equation
- Voltage gated & leak channel



· membrane current



~ Capacitor

전하량 $Q \propto \Delta V$

$\therefore \boxed{Q = C\Delta V}$

Q의 변화량 $\frac{\Delta Q}{\Delta t}$

$$\Rightarrow I_{mem} = \frac{dQ}{dt} = C\frac{dV_{mem}}{dt}$$

- <u>Nernst equation</u> : electrochemical eqn.

$$nFE = nFE^\circ - RT\ln\frac{[Na^+]_{외}}{[Na^+]_{내}}$$

외 내

(derivation) from $\Delta G = \Delta G^\circ - RT\ln Q$ & $\Delta G = W_{비팽창} = nFE$

$$\Rightarrow E_{외} - E_{내} = \frac{RT}{nF}\ln\frac{[Na^+]_{내}}{[Na^+]_{외}} = 0.0592\log_{10}\frac{[Na^+]_{내}}{[Na^+]_{외}} \equiv V_{nerst}$$

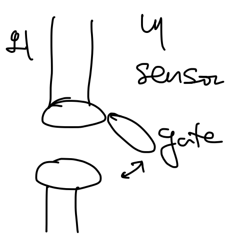
- <u>Voltage gated & leak channel</u>

from $\Delta V = IR$, $I = \frac{\Delta V}{R} \overset{let}{=} g\cdot\Delta V$

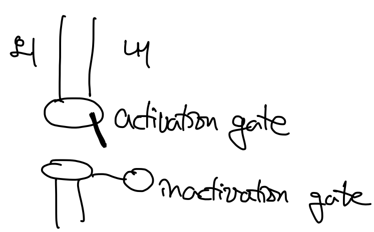
g ← conductance (전기전도도) describes Voltage gated & leak channel

<u>How?</u> Two kinds of conductance (Voltage gated channel)

Persistent conductance (지속적인) (K^+)



Transient conductance (일시적인) (Na^+)



✓ K^+ conductance is constructed w/ four identical subunits.

$$\underbrace{P_K}_{\text{Prob. of channel opening}} = \underbrace{n}_{\text{fraction of opened } K^+ \text{ indep. subunits.}}{}^{4}$$

also varies dep. on voltage.

$$\frac{dn}{dt} = \underbrace{\alpha_n(V)}_{\text{rate constants to open subunit}}(1-n) - \underbrace{\beta_n(V)}_{\text{rate constants to close subunit}}\cdot n$$

$$\left(\times \frac{1}{\alpha_n(V)+\beta_n(V)}\right) \rightsquigarrow \tau_n(V)\frac{dn}{dt} = \underbrace{n_\infty(V)}_{\frac{\alpha_n}{\alpha_n+\beta_n}} - n$$

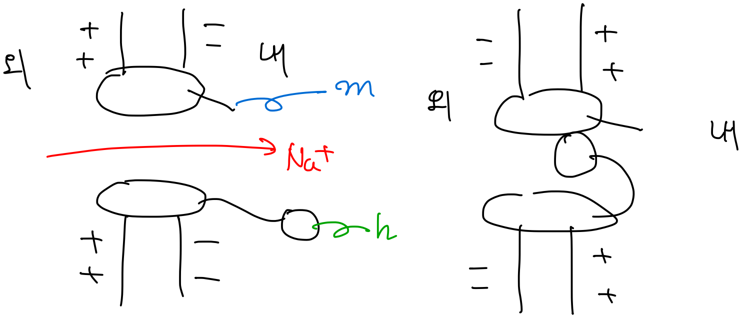
Thermodynamically,

$$\alpha_n(V) \propto \underbrace{e(-aV/k_BT)}_{\text{Boltzmann factor}}$$

$$\beta_n(V) \propto e(-bV/k_BT)$$

→ data fitting

✓ Na^+ Conductance



$$\frac{dm}{dt} = \alpha_m(V)\,(1-m) + \beta_m(V)\,m \quad \text{(or } h\text{)}$$

$$P_{Na} = \underbrace{m^3}_{\text{fraction of opened "activation gate"}} \underbrace{h}_{\text{fraction of "non-activation gate" (opened)}}$$

fraction of "non-activation gate" (opened)

fraction of opened "activation gate"

different voltage dep.

· Hodgkin-Huxley model

$$I_m = C_m \frac{dV_m}{dt} + \bar{g}_K n^4 (V - \underbrace{V_K}_{\text{Nernst}}) + \bar{g}_{Na} m^3 h (V - \underbrace{V_{Na}}_{\text{Nernst}}) + \bar{g}_L (V - \underbrace{V_L}_{\text{leak channel}})$$

Nernst

Nernst

leak channel

Na^+-K^+ pump effect